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MOMENTUM AND ENERGY EXCHANGE IN NONEQUILIBRIUM MULTICOMPONENT MEDIA

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In multicomponent media, the equilibrium states are defined by thermodynamic equilibrium conditions in the form of the equalities between the pressures and temperatures of the components, the maximum entropy principle (or the free-energy minimum) for the mixture, and the equality of the velocities of the components. The conservation laws for the components allow for their interaction with each other in the form of forces and energy fluxes containing the differences of the velocities, pressures, and temperatures of the components. The form of momentum and energy exchange between each component and the continuum expressing the collective properties of the ensemble of components, is also considered. It is shown that these momentum and energy fluxes are different from zero only for the states of multicomponent media with velocity nonequilibrium.

Key words: *equilibrium, multiphase multicomponent medium, interaction of components, closure of the system of equations.*

Introduction. The great diversity of multicomponent media of natural and man-made origins makes their investigation an extremely difficult problem. Dynamic processes in multicomponent media, accompanied by phase transitions of separate components, are especially complex. The components move at different velocities, which leads to variations in their concentrations in the four-dimensional space x_1, x_2, x_3, t. Because of the interaction of the components, a nonequilibrium multicomponent medium reaches an equilibrium state after a certain relaxation time. The relaxation processes of pressures, temperatures, and velocities in multicomponent media have been studied both for particular mixtures and in general formulations [1–7]. In these studies, the sum of the functions defining the momentum and energy exchange between all components was usually set equal to zero. This created difficulties in determining the increase in the entropy of multicomponent media during relaxation. Accounting for nonequilibrium kinetic energy partly eliminates these difficulties.

Below, we consider problems that arise in the development of models for continuum based on the hypothesis of interacting continua [1]. In these models, the components are structural elements of multicomponent media, which are present simultaneously at each point of the volume. Averaging operations using their characteristics make it possible to pass from the characteristics of the components to the characteristics of a certain continuum, which, like the characteristics of the components, are continuous in the four-dimensional space x_1, x_2, x_3, t. Therefore, they can be described by conservation laws. A continuum whose characteristics are obtained by averaging the corresponding characteristics of the mixture components will be referred to as a virtual continuum. Obviously, one of the conditions of equivalence between multicomponent media and corresponding virtual continua is a macrolevel manifestation of the interaction of components in the multicomponent medium. In other words, the conservation laws of a virtual continuum necessarily depend on the conservation laws of the components.

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The main requirements to which models of multicomponent media should satisfy are as follows:

1. Each component with number i ($i = 1, 2, \ldots, N$) is characterized by the volumetric and mass concentrations (α_i and η_i, respectively), which satisfy the conditions

$$\sum_{i=1}^{N} \alpha_i = 1, \qquad \sum_{i=1}^{N} \eta_i = 1. \tag{1}$$

2. For each ith component, there is a complete set of characteristics — the density ρ_i, pressure P_i, temperature T_i, internal energy E_i, entropy S_i, velocity $\boldsymbol{U}_i$, kinetic energy $K_i = 0.5|\boldsymbol{U}_i|^2$, total energy $\varepsilon_i = E_i + K_i$, etc.

3. For each component, the stress tensor is split into a spherical part and a deviator.

4. The thermodynamic quantities (P_i, ρ_i, E_i, T_i, S_i, etc.) are related by equations of state. The modern equations of state [8] describe polymorphic phase transitions, melting, vaporization, and ionization, considerably extending the range of applicability of models of multicomponent media.

5. If the values of P, T and $\boldsymbol{U}$ for components with numbers i and j differ, then momentum and energy exchange occurs between the components.

Continuum of a Component. We shall follow the well-studied approach [2–7] to describing the laws of conservation of components. It is assumed that when multiplied by the volumetric concentration, the specific physical quantities (in unit volume of the ith component) become continuous in the volume occupied by the mixture. They include both the main quantities ($\alpha_i\rho_i$, $\alpha_i P_i$, $\alpha_i\rho_i\boldsymbol{U}_i$, $\alpha_i\rho_i E_i$, $\alpha_i\rho_i K_i$, $\alpha_i\rho_i S_i$, and $\alpha_i\rho_i T_i$) and a number of other combinations of specific (in unit volume) parameters.

We next consider a mixture of media ignoring turbulence, heat conduction, the effect of fields, and chemical reactions. These assumptions for ideal media lead to the simplest laws of conservation of mass, momentum, and energy for the ith component:

$$\frac{\partial}{\partial t}(\alpha_i\rho_i) + \nabla\alpha_i\rho_i\boldsymbol{U}_i = 0; \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\boldsymbol{U}_i) + \frac{\partial}{\partial x_k}(\alpha_i\rho_i U_{ik}\boldsymbol{U}_i) + \nabla\alpha_i P_i = \alpha_i\boldsymbol{R}_i; \tag{3}$$

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\varepsilon_i) + \nabla(\alpha_i\boldsymbol{U}_i(P_i + \rho_i\varepsilon_i)) = \alpha_i\Phi_i. \tag{4}$$

System (2)–(4) is supplemented by the equation of state for the ith component

$$P_i = P_i(\rho_i, E_i) \tag{5}$$

and the equations for the parameters $\boldsymbol{R}_i$ and Φ_i that describe the rate of momentum and energy exchange between the ith component and the remaining components.

We shall consider multicomponent media with possible nonequilibrium with respect to the parameters P, T and $\boldsymbol{U}$. This implies that there are three functions τ_P, τ_T, and τ_U that describe the relaxation times of pressure, temperature, and velocity and depend on the parameters of the interacting components. In real physical processes, the values of τ_P, τ_T, and τ_U are finite. There are, however, a great number of papers devoted to the so-called asymptotic models of multicomponent media, in which all or some of the relaxation times are set equal to zero or infinity. In the present paper, such models are not considered.

Since a multicomponent media is in nonequilibrium, the equations of motion and energy should contain the forces and fluxes generated by particular kinds of nonequilibria, according to the theory of nonequilibrium processes [9]. From this point of view, Eqs. (3) and (4) need to be refined. They include the single force P_i — the spherical part of the stress tensor. One kind of interaction between the components is friction. Therefore, the force F_i exerted on the ith component by the multicomponent medium is taken in the most general form and is considered a tensor. Equation (3) contains the vector $\boldsymbol{R}_i$, which defines the momentum exchange between the ith component and the remaining components of the multicomponent medium. The form of the vector $\boldsymbol{R}_i$ is well justified, but the addition of the tensor force F_i extends the capabilities of the model.

Equation (4) contains the function Φ_i, which defines the energy exchange between the ith component and the remaining components of the multicomponent medium due to nonequilibrium in pressure and temperatures.

We add the energy flux $\boldsymbol{Q}_i$ that depends on the nonequilibrium in velocities. After the introduction of the force F_i and the flux $\boldsymbol{Q}_i$, the equations of motion (3) and energy (4) are written as

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\boldsymbol{U}_i)+\frac{\partial}{\partial x_k}(\alpha_i\rho_i U_{ik}\boldsymbol{U}_i)+\nabla\alpha_i P_i+\frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik})=\alpha_i\boldsymbol{R}_i; \tag{6}$$

$$\frac{\partial}{\partial t}(\alpha_i\rho_i\varepsilon_i)+\nabla(\alpha_i\boldsymbol{U}_i(P_i+\rho_i\varepsilon_i))+\frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik}\boldsymbol{U}_i)+\nabla\alpha_i\boldsymbol{Q}_i=\alpha_i\Phi_i. \tag{7}$$

Exchange Terms $\boldsymbol{R}_i$ and $\boldsymbol{\Phi_i}$. In the overwhelming majority of papers (see, for example, [2–7]) it is assumed that the momentum and energy exchange between the ith and jth components of a multicomponent medium is defined by the vector $\boldsymbol{R}_{ij}$ and the function Φ_{ij}. From the conservation laws it follows that

$$\boldsymbol{R}_{ij}=-\boldsymbol{R}_{ji},\qquad \Phi_{ij}=-\Phi_{ji}. \tag{8}$$

It is assumed in this case that a component does not interact with itself and that $\boldsymbol{R}_{ii}=0$ and $\Phi_{ii}=0$. Let us consider the most widely used forms of $\boldsymbol{R}_{ij}$ and Φ_{ij}:

$$\boldsymbol{R}_{ij}=a_{ij}(\boldsymbol{U}_j-\boldsymbol{U}_i); \tag{9}$$

$$\Phi_{ij}=\varphi_{ij}(P_j-P_i)+\psi_{ij}(T_j-T_i). \tag{10}$$

The functions a_{ij}, φ_{ij}, and ψ_{ij} have a particular form that depends on the state of aggregation and phase states of the ith and jth components, the sizes, shape, and surface roughness of the particles, and the mechanical and thermal properties of the components. In order that conditions (8) be fulfilled, the functions a, φ, and ψ should satisfy to Onsager reciprocal relations [9]

$$a_{ij}=a_{ji},\qquad \varphi_{ij}=\varphi_{ji},\qquad \psi_{ij}=\psi_{ji}. \tag{11}$$

The total momentum and total energy acquired (lost) by the ith component when exchanging with the remaining components of the mixture are defined by the equations

$$\boldsymbol{R}_i=\sum_{j=1}^{N}\alpha_j\boldsymbol{R}_{ij},\qquad \Phi_i=\sum_{j=1}^{N}\alpha_j\Phi_{ij}. \tag{12}$$

Let us consider one of the possible functions a_{ij} that satisfy conditions (11), namely,

$$a_{ij}=a\rho_i\rho_j. \tag{13}$$

Substituting (13) into (9) and summing over j according to (12), we obtain the following expression for the vector $\boldsymbol{R}_i$:

$$\boldsymbol{R}_i=a\rho_i\Big(\sum_{j=1}^{N}\alpha_j\rho_j\boldsymbol{U}_j-\boldsymbol{U}_i\sum_{j=1}^{N}\alpha_j\rho_j\Big). \tag{14}$$

Using the widely used averaging rules

$$\rho=\sum_{i=1}^{N}\alpha_i\rho_i, \tag{15}$$

$$\rho\boldsymbol{U}=\sum_{i=1}^{N}\alpha_i\rho_i\boldsymbol{U}_i, \tag{16}$$

we reduce expression (14) to the form

$$\boldsymbol{R}_i=a\rho_i\rho(\boldsymbol{U}-\boldsymbol{U}_i). \tag{17}$$

The velocity $\boldsymbol{U}$ defined by Eq. (16) is called the mass-averaged or barycentric velocity.

We pay attention that the forms (14) and (17) are equivalent. However, Eq. (14) defines the interaction of the ith component with the multicomponent medium by taking into account the interaction with each jth component, and expression (17) defines the interaction of the ith component with a virtual continuum whose properties are determined by the average values of ρ and $\boldsymbol{U}$. Thus, the virtual continuum is a participant of the exchange. If the virtual continuum is assigned the subscript 0, relation (17) becomes

$$\boldsymbol{R}_{i0} = a\rho_i\rho_0(\boldsymbol{U}_0 - \boldsymbol{U}_i).$$

It is easy to show that this vector satisfies condition (8) in the form

$$\boldsymbol{R}_{i0} = -\boldsymbol{R}_{0i}. \tag{18}$$

By analogy with $\boldsymbol{R}_{ij}$, we consider the rate of energy exchange (10) between the ith and jth components:

$$\Phi_{ij} = \varphi(P_j - P_i) + \psi\rho_i\rho_j(T_j - T_i). \tag{19}$$

Summing over j according to (12), we obtain

$$\Phi_i = \varphi\Big(\sum_{j=1}^{N}\alpha_j P_j - P_i\sum_{j=1}^{N}\alpha_j\Big) + \psi\rho_i\Big(\sum_{j=1}^{N}\alpha_j\rho_j T_j - T_i\sum_{j=1}^{N}\alpha_j\rho_j\Big). \tag{20}$$

Using (1) and (15) and the averaging rules

$$P = \sum_{i=1}^{N}\alpha_i P_i, \tag{21}$$

$$\rho T = \sum_{i=1}^{N}\alpha_i\rho_i T_i, \tag{22}$$

we write relation (20) as the rate of energy exchange between the virtual continuum and the ith component:

$$\Phi_i = \varphi(P - P_i) + \psi\rho_i\rho(T - T_i). \tag{23}$$

If the parameters of the virtual continuum are assigned the subscript $j = 0$, Eq. (23) becomes

$$\Phi_{i0} = \varphi(P_0 - P_i) + \psi\rho_0\rho_i(T_0 - T_i). \tag{24}$$

From relations (19), (20), (23), and (24), it is obvious that condition (8) is satisfied, including for $j = 0$:

$$\Phi_{i0} = -\Phi_{0i}. \tag{25}$$

The aforesaid implies that if the ith component acquires a certain momentum or energy, the virtual continuum transfers the same momentum or energy to it. In other words, the conservation laws of the continuum should contain the corresponding terms that summarize all momenta and energies transferred from the virtual continuum to the components.

The results obtained above are of fundamental importance. They show that the relaxation fluxes of momentum and energy in a multicomponent medium can be treated as the sum of exchanges between all pairs of components or as the exchange between each component and the virtual continuum, which expresses the properties of the multicomponent medium, i.e., all components.

For the chosen form of the functions a_{ij}, φ_{ij}, and ψ_{ij}, the vector $\boldsymbol{R}_i$ and the function Φ_i are such that their sums over all $i = 1, 2, \ldots, N$ vanish:

$$\boldsymbol{R} = \sum_{i=1}^{N}\alpha_i\boldsymbol{R}_i = 0, \qquad \Phi = \sum_{i=1}^{N}\alpha_i\Phi_i = 0 \tag{26}$$

or, what is the same,

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\boldsymbol{R}_{ij} = 0, \qquad \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j\Phi_{ij} = 0.$$

Below, we restrict ourselves to an analysis of models containing $\boldsymbol{R}_{ij}$ and Φ_{ij} that satisfy (8), (11), and (26).

Continuum of a Mixture. Equations (2) and (5)–(7) are obtained after passage from the microlevel to the macrolevel for the ith component. However, the components of a multicomponent medium are its structural elements. In other words, the classical averaging methods allow one to pass from a structural medium at the microlevel to a structural medium at the mesolevel.

To pass from a structural multicomponent medium to the corresponding virtual continuum, it is customary to employ (see [1–7]) equations that relate the characteristics of the components to the averaged characteristics of the multicomponent medium. These are Eqs. (15), (16), (21), and (22), and the relations

$$\rho E = \sum_{i=1}^{N} \alpha_i \rho_i E_i. \tag{27}$$

The averaging of the specific total energy ε_i requires a close consideration. The fact is that $E = \varepsilon - \boldsymbol{U}\boldsymbol{U}/2$ and $E_i = \varepsilon_i - \boldsymbol{U}_i\boldsymbol{U}_i/2$, and if we substitute these quantities into (27), we obtain

$$\rho\varepsilon = \sum_{i=1}^{N} \alpha_i \rho_i (\varepsilon_i - H_i), \tag{28}$$

where

$$H_i = (\boldsymbol{U}_i\boldsymbol{U}_i - \boldsymbol{U}\boldsymbol{U})/2. \tag{29}$$

The quantity H_i is called the nonequilibrium kinetic energy of the ith component. If we introduce the nonequilibrium kinetic energy H

$$\rho H = \sum_{i=1}^{N} \alpha_i \rho_i H_i, \tag{30}$$

then, relation (28) becomes

$$\rho(\varepsilon + H) = \sum_{i=1}^{N} \alpha_i \rho_i \varepsilon_i. \tag{31}$$

Multicomponent media are described by the systems of conservation laws and equations of state for the components. Their number is equal to the number of the components N. Solving all these systems, we obtain the quantities ρ_i, P_i, $\boldsymbol{U}_i$, T_i, E_i, α_i, etc. ($i = 1, 2, \dots, N$) as functions of x_1, x_2, and x_3 for a certain time t. Next, we average these quantities using Eqs (15), (16), (21), (22), and (27)–(31) and obtain the characteristics

$$\rho, \quad P, \quad \boldsymbol{U}, \quad E, \quad T, \quad \dots . \tag{32}$$

The problem at which we arrived can be formulated as follows: What the system of conservation laws for the virtual continuum should be in order that its solution coincides with (32)?

In the classical continuum model, the macrocharacteristics of materials P, ρ, E, $\boldsymbol{U}$, T, S, etc., are obtained after averaging of the characteristics of a large number of microparticles. A description of the averaging rules is beyond the scope of the present paper and are given in detail in [10–13]. It is important that the average characteristics of materials (macroquantities) are continuous in the space of x_1, x_2, x_3, t and can be described by conservation laws, which are closed by equations of state in the form of relations between thermodynamic quantities. It is known, however, that the equations of state describe the equilibrium states of materials. For the case of nonequilibrium states, the conservation laws should contain additional forces and fluxes that describe the relaxation of nonequilibrium states to equilibrium states [9]. The conservation laws for the macroquantities of a continuum ignoring turbulence, chemical reactions, the effects of fields (electromagnetic, gravity, etc.), and heat conduction but taking into account tensor forces, energy fluxes, and exchange terms on the right sides of the equations have the form

$$\frac{\partial \rho}{\partial t} + \nabla \rho \boldsymbol{U} = 0; \tag{33}$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{U}) + \frac{\partial}{\partial x_k}(\rho U_k \boldsymbol{U}) + \nabla P + \frac{\partial}{\partial x_k}(\boldsymbol{F}_k) = \boldsymbol{R}; \tag{34}$$

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \nabla \boldsymbol{U}(P + \rho\varepsilon) + \frac{\partial}{\partial x_k}(\boldsymbol{F}_k \boldsymbol{U}) + \nabla \boldsymbol{Q} = \Phi. \tag{35}$$

System (33)–(35) is not closed by the equation of state since the functions ρ, P, $\boldsymbol{U}$, E, and ε are not related by an equation of state and are the averaged characteristics of the virtual continuum defined by Eqs. (15), (16), (21), (22), and (27)–(31).

Relationship between the Conservation Laws of the Macrolevels and Mesolevels. There have been repeated attempts to pass from the mesolevel in describing multicomponent media to the macrolevel. In almost all papers (see [2, 4, 5]), Eqs. (2)–(4) were summed so as to obtain Eqs. (33)–(35). We proceed in the opposite way, bearing in mind that the characteristics of the continuum are introduced by Eqs. (15), (16), (21), (22), and (27)–(31), and the equations to which they should satisfy need to be found. In this case, it is not obvious beforehand whether the capabilities provided by (33)–(35) will suffice for this purpose.

We substitute ρ and $\rho\boldsymbol{U}$ from (15) and (16) into the law of conservation of mass (33). As a result, we obtain

$$\sum_{i=1}^{N}\Big(\frac{\partial}{\partial t}(\alpha_i\rho_i)+\nabla\alpha_i\rho_i\boldsymbol{U}_i\Big)=0. \qquad (36)$$

Each term in (36) is equal to zero since it coincides with the law of conservation of mass for the ith component (2).

Similarly, into the law of conservation of momentum (34), we substitute relations (21) and (26), and expression (16) in the form

$$\rho U_k=\sum_{i=1}^{N}\alpha_i\rho_i U_{ik}, \qquad (37)$$

and the relation

$$\boldsymbol{F}_k=-\sum_{j=1}^{N}\alpha_i\boldsymbol{F}_{ik} \qquad (38)$$

obtained using the averaging rules and (18). Then, Eq. (34) becomes

$$\sum_{i=1}^{N}\Big(\frac{\partial}{\partial t}(\alpha_i\rho_i\boldsymbol{U}_i)+\frac{\partial}{\partial x_k}(\alpha_i\rho_i U_{ik}\boldsymbol{U})+\nabla\alpha_i P_i-\frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik})-\alpha_i\boldsymbol{R}_i\Big)=0. \qquad (39)$$

The condition of coincidence of each term in (39) with Eq. (6) has the form

$$\frac{\partial}{\partial x_k}(2\alpha_i\boldsymbol{F}_{ik}-\alpha_i\rho_i U_{ik}(\boldsymbol{U}-\boldsymbol{U}_i))=0.$$

After integration, we arrive at the following expression for the components of the force F_i

$$\boldsymbol{F}_{ik}=\rho_i U_{ik}(\boldsymbol{U}-\boldsymbol{U}_i)/2. \qquad (40)$$

The integration constant is determined from the condition $\boldsymbol{F}_{ik}=0$ for $\boldsymbol{U}=\boldsymbol{U}_i$.

Let us now consider the energy equation (35). We substitute into it the quantity Φ, which satisfies Eqs. (21), (26), (28), and (38), and the quantity $\boldsymbol{Q}$ expressed in terms of $\boldsymbol{Q}_i$ with allowance for (25), namely,

$$\boldsymbol{Q}=-\sum_{i=1}^{N}\alpha_i\boldsymbol{Q}_i. \qquad (41)$$

As a result, we obtain the energy conservation law for the virtual continuum

$$\sum_{i=1}^{N}\Big(\frac{\partial}{\partial t}(\alpha_i\rho_i(\varepsilon_i-H_i))+\nabla(\alpha_i\boldsymbol{U}(P_i+\rho_i\varepsilon_i-\rho_i H_i))-\frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik}\boldsymbol{U})-\nabla\alpha_i\boldsymbol{Q}_i-\alpha_i\Phi_i\Big)=0. \qquad (42)$$

The condition of coincidence of each term in (42) with the energy equation ith of a component (7) has the form

$$\frac{\partial}{\partial t}(\alpha_i\rho_i H_i)+\nabla\alpha_i(P_i+\rho_i\varepsilon_i)(\boldsymbol{U}_i-\boldsymbol{U})+\nabla\alpha_i\rho_i\boldsymbol{U}H_i+\frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik}(\boldsymbol{U}_i+\boldsymbol{U}))+2\nabla\alpha_i\boldsymbol{Q}_i=0. \qquad (43)$$

We assume that the energy flux $\boldsymbol{Q}_i$ is expressed in terms of the functions describing the state of the ith component as follows:

$$\boldsymbol{Q}_i = (P_i + \rho_i \varepsilon_i)(\boldsymbol{U} - \boldsymbol{U}_i)/2. \tag{44}$$

In this case, Eq. (43) becomes

$$\frac{\partial}{\partial t}(\alpha_i \rho_i H_i) + \nabla \alpha_i \rho_i H_i \boldsymbol{U} + \frac{\partial}{\partial x_k}(\alpha_i \boldsymbol{F}_{ik}(\boldsymbol{U}_i + \boldsymbol{U})) = 0. \tag{45}$$

Substituting the expression of $\boldsymbol{F}_{ik}$ from (40) into (45) and taking into account relation (29), we obtain the following equation for the nonequilibrium kinetic energy H_i:

$$\frac{\partial}{\partial t}(\alpha_i \rho_i H_i) + \frac{\partial}{\partial x_k}(\alpha_i \rho_i H_i (U_k - U_{ik})) = 0. \tag{46}$$

Completeness of the Model. System (2)–(5), which is widely used to describe the behavior of the ith component of multicomponent media, together with the equation $\varepsilon_i = E_i + \boldsymbol{U}_i \cdot \boldsymbol{U}_i/2$, is not closed since it contains seven equations for the required eight functions (P_i, ρ_i, E_i, ε_i, U_{1i}, U_{2i}, U_{3i}, and α_i). In contrast to the classical conservation laws for a continuum, the above equations contain the volumetric concentration α_i. The fact that system (2)–(5) is unclosed generates a large number of particular models that assume the absence of nonequilibrium in pressure, temperature or velocity. The assumption of partial equilibrium of multicomponent media makes the system closed but reduces the generality and the region of applicability of the model. It was shown above that if the force component F_i defined by Eq. (40) and the energy flux $\boldsymbol{Q}_i$ defined by Eq. (44) are introduced into the equation for the ith component, system (2), (6), (7) is supplemented by the equation for the nonequilibrium kinetic energy (46) and the expression of H_i in terms of $\boldsymbol{U}_i$ and $\boldsymbol{U}$ from (29). Thus, the new system of equations for the ith component contains nine equations for nine required functions and, hence, it is closed.

The force F_i and the energy flux $\boldsymbol{Q}_i$ proposed in the present paper are not equal to zero only in the case of nonequilibrium in the velocity $\boldsymbol{U}$. They are directly proportional to the difference between the barycentric velocity $\boldsymbol{U}$ and the individual velocity of the ith component $\boldsymbol{U}_i$. In the case of equilibrium in $\boldsymbol{U}$, the quantities F_i and $\boldsymbol{Q}_i$ vanish. We note one feature of the introduced parameters F_i and $\boldsymbol{Q}_i$. They do not contain empirical functions or constants. The individual features of each component (surface roughness, particle size, sound velocity, etc.) still should be allowed for by the functions a, φ, and ψ in the expressions for $\boldsymbol{R}_i$ and Φ_i.

It was shown above that in the case of using the averages (15) and (16), summation of the laws of conservation of mass for the components (2) results in the law of conservation of mass for the virtual continuum (33). Let us sum Eq. (6):

$$\frac{\partial}{\partial t}\sum_{i=1}^{N} \alpha_i \rho_i \boldsymbol{U}_i + \frac{\partial}{\partial x_k}\sum_{i=1}^{N} \alpha_i \rho_i U_{ik} \boldsymbol{U}_i + \nabla \sum_{i=1}^{N} \alpha_i P_i + \frac{\partial}{\partial x_k}\sum_{i=1}^{N} \alpha_i \boldsymbol{F}_{ik} = \sum_{i=1}^{N} \alpha_i \boldsymbol{R}_i. \tag{47}$$

Using Eqs. (16), (21), and (26), we bring Eq. (47) to the form

$$\frac{\partial}{\partial t}\rho \boldsymbol{U} + \frac{\partial}{\partial x_k}\sum_{i=1}^{N} \alpha_i \rho_i U_{ik} \boldsymbol{U}_i + \nabla P + \frac{\partial}{\partial x_k}\sum_{i=1}^{N} \alpha_i \boldsymbol{F}_{ik} = 0. \tag{48}$$

From Eq. (40) it follows that

$$\rho_i U_{ik} \boldsymbol{U}_i + \boldsymbol{F}_{ik} = \rho_i U_{ik} \boldsymbol{U} - \boldsymbol{F}_{ik}. \tag{49}$$

We use (37), (38), and (49) and reduce Eq. (48) to the form (34). Therefore, summation over all components of the laws of conservation of momentum for the ith component yields the law of conservation of momentum for the virtual continuum.

We make similar manipulations with the energy equation. Before summation of Eq. (7) over i, we bring it, using the equation

$$\boldsymbol{U}_i(P_i + \rho_i \varepsilon_i) + \boldsymbol{Q}_i = \boldsymbol{U}(P_i + \rho_i \varepsilon_i) - \boldsymbol{Q}_i$$

[which follows from (44)] to the form

$$\frac{\partial}{\partial t}(\alpha_i \rho_i \varepsilon_i) + \nabla(\alpha_i \boldsymbol{U}(P_i + \rho_i \varepsilon_i)) + \frac{\partial}{\partial x_k}(\alpha_i \boldsymbol{F}_{ik} \boldsymbol{U}_i) - \nabla \alpha_i \boldsymbol{Q}_i = \alpha_i \Phi_i.$$

After summation and using the equations of passage from the sums to the macroquantities (21), (26), (31), and (41), we obtain

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \nabla\boldsymbol{U}(P+\rho\varepsilon) + \sum_{i=1}^{N}\Big(\frac{\partial}{\partial t}(\alpha_i\rho_i H_i) + \nabla(\alpha_i\rho_i H_i\boldsymbol{U})$$

$$+ \frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik}(\boldsymbol{U}_i+\boldsymbol{U})) - \frac{\partial}{\partial x_k}(\alpha_i\boldsymbol{F}_{ik}\boldsymbol{U})\Big) + \nabla\boldsymbol{Q} = 0.$$

The expression under the summation sign is transformed to (38) and (45) by means of (35). Thus, the summation of the energy conservation laws for the components results in the energy conservation law for the virtual continuum.

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REFERENCES

1. Kh. A. Rakhmatulin, "Foundations of the gas dynamics of mutually penetrating motions of compressible media," *Prikl. Mat. Mekh.*, **20**, No. 2, 184–195 (1956).
2. A. N. Kraiko, R. I. Nigmatulin, V. K. Starkov, and L. E. Sternin, "Mechanics of multiphase media," *Itogi Nauki Tekh., Gidromekhanika*, **6**, 93–174 (1973).
3. S. S. Kutateladze and M. A. Styrikovich, *Hydraulics of Gas–Liquid Systems* [in Russian], Gosénergoizdat, Moscow (1958).
4. R. I. Nigmatulin, *Foundations of the Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
5. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Parts 1 and 2, Hemisphere, New York (1991).
6. V. F. Kuropatenko, "Unsteady flows of multicomponent media," *Mat. Model.*, **1**, No. 2, 118–136 (1989).
7. N. N. Yanenko, R. I. Soloukhin, A. N. Papyrin, and V. M. Fomin, *Supersonic Two-Phase Flows Under Conditions of Particle Velocity Nonequilibrium* [in Russian], Nauka, Novosibirsk (1980).
8. V. F. Kuropatenko, "Equations of state in mathematical models of mechanics and physics," in: *Extreme States of Matter* (collected scientific papers) [in Russian], Inst. Comput. Technol., USSR Acad. of Sci. (1991), pp. 3–38.
9. S. R. De Groot, *Thermodynamics of Irreversible Processes,* North Holland Publishing Company, Amsterdam (1952).
10. J. O. Hirschfelder, C. F. Curtis, and R. B. Bird, *Molecular Theory of Gases and Liquids,* Wiley, New York (1954).
11. H. S. Tsien, *Physical Mechanics* [Russian translation], Mir, Moscow (1965).
12. Ya. I. Frenkel, *Statistical Physics* [in Russian], Izd. Akad. Nauk SSSR, Moscow–Leningrad (1948).
13. F. Daniels and R. A. Alberty, *Physical Chemistry*, Wiley, New York (1963).